GEOMETRICAL SOLUTION OF PROB. 126. (*P.* 188, *VOL. III.*)

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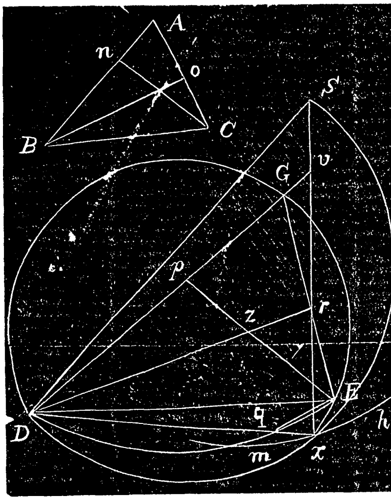
Analysis. Let ABC be the required $\triangle$, the bisectors Bo, Cn of the ang's B and C are given, and the vertical angle A, to construct the triang. ABC.



Take DE a given line $= 2Cn$ say, and upon it describe a segment of a circle DGE containing an angle $= BAC$; this segm't is .˙. given in position and magnitude. Conceive the $\triangle DGE$ to be similar to the $\triangle ABC$, and bisect the angles D, E; .˙. the ratio of $Dr : Ep$ is given being the same as $Bo : Cn$.

Make angle $EDS = Ezr$, which is a given angle (being half of the supplement of angle G), .˙. DS is in posit'n. Draw rS, making the angle $DrS =$ the angle EpD, and .˙. the angles DEp, DrS are similar and $ED:DS::Ep:Dr$, in a given ratio, and ED is given; .˙. DS is given and S is .˙. a given point.

Produce DG to v and draw Dx making angle $vDx = EDS = Ezr$, a given angle, and produce Sr to meet Dx in the point x; and as angle $DrS = Epd$, .˙. $Ezr = rvp =$ a given angle (the quadrilateral $vpzr$ is inscriptable in a circle), and as vDx is a given angle, .˙. Dxv is a given angle, and the line DS is given in position and magnitude; .˙. the seg't DxS is given.

Draw Eq making angle $DEq = DSv$, and as $EDS = vDx$, .˙. ang. $SDv = EDq$; hence the $\triangle$s DSv, DEq are similar, and .˙. ang. $DqE = DvS$, a given angle; .˙. as DE is given, the segment DqE is given in postion and magnitude. Now $SD : DE :: vD : Dq$, that is, in a given ratio, and the ratio of $vD : Dx$ is also given (as the $\triangle Dvx$ has all its angles given); .˙. the ratio of $Dx : Dq$ is given and D a given point, and q on the circumf. of a given circle; .˙. the locus of x is a circle, mxh, given in position and magnitude. (See Chauvenet's Geom., p. 314.) .˙. the point x is given, and Dx is in position; and the angle xDv being given, Dv is in pos'n when the point G is given; .˙. the $\triangle DGE$ is given and hence $\triangle ABC$ is given.

The construction follows readily from the analysis; but the calculation of the sides is rather long.